Lab 5: The derivative at a point – part 1

Lab Preparation: Answer the following questions individually and bring your write-up to class.

A. Draw a full-page picture of the physical context in three different configurations (specified in the context description) overlayed. This will help illustrate how the relevant quantities are changing in the region of interest. You will redraw and add to this picture during class.

B. What happens to the changes in the dependent quantity (volume, gravitational force, or mass of Iodine-123) as the independent quantity (radius, separation distance, or time) is incremented by constant amounts? Is your rate of change constant, increasing or decreasing?

C. Draw a full-page graph showing the relationship between the two quantities involved in the instantaneous rate that you are asked to approximate. Add a point for each of the configurations you drew in your picture from A. Represent the changes in both quantities on your graphs as the length of short line segments. You will redraw and add to this graph during class.

D. On your picture and your graph illustrate and label the changes in the relevant quantities to support your answer to B (using both Δ-notation and numerical values).

Lab Instructions: Work with your group on the problem assigned to you. We encourage you to collaborate both in and out of class, but you must write up your responses individually.

1. Compare your picture of your physical situation with the others in your group. Using the best ideas from your group, redraw a full-page picture of several overlayed snapshots showing
   a. The system at the moment for which the instantaneous rate is requested
   b. Other configurations from your lab preparation
   c. Changes in relevant quantities labeled (using both Δ-notation and numerical values) to support your answers in the lab preparation.

   You will return to your picture to include additional information.

2. Compare your graph showing the relationship between the two quantities in your rate with the others in your group. Using the best ideas from your group, redraw a full-page graph showing
   a. Several points corresponding to the moment for which the instantaneous rate is requested and three or four other configurations
   b. Algebraic and numerical representations of the quantities at the points included in Part a
   c. Changes in the quantities starting from the moment at which you need to determine the instantaneous rate (using both Δ-notation and numerical values)

   You will return to your graph to include additional information.
3. In this question, you will provide details about what you have been asked to approximate.
   a. Describe what you have been asked to approximate using language from your context.
   b. Define a variable to represent the unknown value algebraically. What units will be attached to it?
   c. Represent the unknown value that you are approximating on your graph. Label it with your chosen variable. What attribute of the object that you added to your graph corresponds to the unknown value you are approximating?

4. In this question, you will provide details about approximations to your instantaneous rate.
   a. Compute 3 average rates of change that approximate the requested instantaneous rate.
   b. Using language about your context, explain the physical meaning of one of your average rates of change.
   c. Write an algebraic expression showing someone how to compute these average rates in general.
   d. Represent the 3 approximations on your graph. Label them with numerical values. What attribute of the things that you added to your graph corresponds to the approximation values?

5. In this question, you will identify both underestimates and overestimates for the requested instantaneous rate.
   a. If you have not already found both underestimates and overestimates and represented them on your graph, do so.
   b. Using only language about your physical context (not your graph), explain how you know these are in fact underestimates and overestimates.
   c. Explain how your explanation from Part b can be seen on both the picture of the situation and on the graph.

6. In this question, you will identify and represent the errors in your approximations.
   a. Give an algebraic representation of the errors for both an underestimate and an overestimate.
   b. Explain how these errors are represented graphically. Add and label the errors on your graph.

7. In this question, you will identify and represent the error bounds in your approximations.
   a. Find an error bound for one of your approximations. Justify your answer.
   b. Explain how this error bound is represented graphically. Add and label the error bound on your graph.
   c. What is the resulting range of possible values for your instantaneous rate? Explain how this range is represented graphically.
Lab 5: The Derivative at a Point – Part 1

In-Class Context: A bolt is fired from a crossbow straight up into the air with an initial velocity of 49 m/s. Accounting for wind resistance proportional to the speed of the bolt, its height above the ground is given by the equation \( h(t) = 7350 - 245t - 7350e^{-t/25} \) meters (with \( t \) measured in seconds). Approximate the speed when \( t = 2 \) seconds accurate to within 0.1 m/s.

Context 1: Approximate the instantaneous rate of change of the volume of a sphere with respect to its radius when the radius is 5 cm.
Initial configurations to represent: radius = 3.5 cm, 5 cm, and 6.5 cm

Context 2: NASA has determined that asteroid 1999 RQ36 has a 1 in 1000 chance of colliding with Earth on September 24, 2182*. The force of gravity, \( g \), in Newtons (N) between two objects is inversely proportional to the square of the distance, \( s \) in meters (m), separating them. The constant of proportionality is \( GMm \) where \( G \) is the “universal constant of gravity” \( 6.67 \times 10^{-11} \) Nm²/kg² while \( M = 5.97 \times 10^{24} \) kg and \( m = 1.4 \times 10^{11} \) kg are the masses of the earth and the asteroid, respectively. Approximate the instantaneous rate of change of the gravitational force between the Earth and 1999 RQ36 with respect to distance when the two objects are 10,000,000 m apart.
Initial configurations to represent: \( s = 8 \times 10^6 \), \( 1 \times 10^7 \), and \( 1.2 \times 10^7 \)
Suggested graphing window: \( 5 \times 10^6 \leq s \leq 1.5 \times 10^7 \) and \( 0 \leq g \leq 2.5 \times 10^{12} \)

Context 3: The half-life of Iodine-123, used in medical radiation treatments, is about 13.2 hours. Approximate the instantaneous rate at which the Iodine-123 is decaying 5 hours after a dose of 6.4 \( \mu \)g is administered.
Initial configurations to represent: after 5 hours, 10 hours, and 15 hours

*Class is canceled on September 24, 2182.