Lab 8: Quadratic Approximation

The NASA Q36 Robotic Lunar Rover can travel up to 3 hours on a single charge and has a range of 1.6 miles. After leaving its base and traveling for $t$ hours, the speed of the Q36 is given by the function $v(t) = \sin \sqrt{9 - t^2}$ in miles per hour. One hour into an excursion, the Q36 will have traveled 0.19655 miles. Two hours into a trip, the Q36 will have traveled 0.72421 miles.

In Lab 7, you used tangent lines to approximate the distance $x(t)$ traveled by the Q36 Rover. Lines with slope $m$ through the point $(t_0, x_0)$ can be written in point-slope form as $x = x_0 + m(t - t_0)$. You used the derivative $v(t) = x'(t)$ to find the slope at $t_0$.

We could improve our approximations by using “best fit parabolas.” For the following problems, note that $x = x_0 + a(t - t_0) + b(t - t_0)^2$ is the equation of a parabola that passes through the point $(t_0, x_0)$. Changing the parameters $a$ and $b$ will change the shape of the parabola without changing the fact that it passes through that point.

**Lab Preparation:** Answer the following questions individually and bring your write-up to class.

a. Sketch a parabola on your large graph of $x(t)$ that you think represents the best fit parabola at time $t_0 = 1$. Then determine the equation of this parabola using the form $x = x_0 + a(t - t_0) + b(t - t_0)^2$ and the point $(t_0, x_0) = (1, 0.19655)$. To do this, find the first and second derivatives of the equation for this parabola, set $x'(1) = v(1)$ and $x''(1) = v'(1)$, then solve for $a$ and $b$.

b. Sketch a parabola on your large graph of $x(t)$ that you think represents the best fit parabola at time $t_0 = 2$. Then determine the equation of this parabola using the form $x = x_0 + a(t - t_0) + b(t - t_0)^2$ and the point $(t_0, x_0) = (2, 0.72421)$. To do this, find the first and second derivatives of the equation for this parabola, set $x'(2) = v(2)$ and $x''(2) = v'(2)$, then solve for $a$ and $b$.

**Lab:** Verify that everyone in your group found the same quadratic approximations at $t_0 = 1$ and $t_0 = 2$, then answer the following questions. We encourage you to collaborate both in and out of class, but you must write up your responses individually.

1. Use your quadratic equation at time $t_0 = 1$ to find a more accurate approximation to the distance traveled in the 10 minutes after the $t_0 = 1$ hour mark (i.e., more accurate than your linear approximation in Lab 7). Then find an error bound for this approximation.

   The error bound for a quadratic approximation centered at $t_0$ and evaluated at $t$ is
   \[
   E(t) \leq \frac{M}{6} |t - t_0|^3 \text{ where } M = \max |x''| \text{ between } t \text{ and } t_0.
   \]

   (I recommend graphing $x'''$ to determine a value for $M$.)

2. Use your quadratic equation at time $t_0 = 2$ to find a more accurate approximation to the time the rover reaches the 0.75 mile mark (i.e. more accurate than your linear approximation in Lab 7).