Lab 11: Related Rates

Lab Preparation: For both of your group’s problem,

a. Draw a diagram with all relevant constants and variables labeled.

b. Identify the rates (as derivatives of variables that you identified) that will be used in the problem.

The stories you are about to read about your classmates are true; only the names have been changed to protect the innocent.

1. Luke got a new sand box over the weekend and took his “friends,” He-Man and Skeletor, out to play make-believe.
   a. Skeletor tied He-Man to a pole and began dumping sand on top of him at a rate of 4 cubic inches per second. He-Man is six inches tall. At the moment he is half buried, He-Man notices that the sand is rising at a rate of 1/2 inch per second. How much longer does Lu... I mean He-Man have to come up with a way to escape before he is completely buried?
   b. Annette show up with Batgirl at the new sand box. Batgirl says that burying He-Man is boring and she would rather do Calculus! Help Batgirl, and Skeletor do the following problem from one of their professor’s old exams:
      While Violet Beauregarde is visiting Willy Wonka’s Chocolate Factory, she snatches some experimental gum against Wonka’s warnings. Not yet perfected, the gum has the side effect of expanding Violet into a giant human blueberry. At one point, after she becomes perfectly spherical, she reaches a volume of 1.8 m$^3$ and is still growing at 0.2 m$^3$/s. How fast is her surface area increasing?

2. a. The speed limit on a straight stretch of highway is 55 mph. Lucy, a highway patrol officer, stations herself at a point out of view of the motorists 50 feet off the highway. She is equipped with a radar gun which measures the speed at which a car approaches her position. She takes a reading of suspected speeders by pointing her radar gun at a point on the highway 120 feet from the point on the highway closest to her. The radar gun picks up a reading of 48 feet/sec for a green Chevy driven by Bob and Mark returning to campus from an epic road trip. Should they get a ticket? Justify.
   b. A streetlight hangs 5 meters above the ground. Landen has the ability to magically shrink herself. At what rate must she do this to keep her shadow a constant length of 3 meters while walking away from the light at a speed of 2 m/s?
3. a. Inspired by recently renting “Saturday Night Fever” Art and Sam are redecorating their 16’x12’ living room in a disco theme. Kelli stops by to drop off a ladder they had asked to borrow, but when Tim opens the door, she is shocked by the sight of a disco ball rotating once every 2 seconds from the center of the ceiling. Her horror is replaced by a trance-like state as she is hypnotized from tracking one of the spots of light spinning around the room. As this spot of light enters a corner going from a long wall to a short wall, how fast is it moving?

b. Assume that Lou is perfectly spherical and that he melts at a rate proportional to his surface area, \( A \) (i.e., \( \frac{dV}{dt} = kA \) for some negative constant \( k \)) how fast is Lou’s radius changing when it passes the 3 cm mark? when his radius is 5 cm? when his radius is \( r \) cm? (Your answers might involve the constant \( k \).)

4. a. Andrew and Miguel are on their annual hunting trip. This time, however, they plan on outsmarting the deer! Andrew is sitting to Miguel’s right (east) when the perfect buck appears 40 meters to the north. Miguel aims, but Andrew sneezes an inhumanly loud sneeze. The deer startles and takes off straight southeast at 13 meters per second. Miguel turns to keep the deer centered in his sight, but can’t get a clean shot. At the instant Miguel smacks Andrew in the head with the barrel of the rifle, how fast was he rotating?

b. Elizabeth has the ability to walk on vertical surfaces, and is walking up and down a 5 meter tall lamp post at a rate of 1.5 m/s. Remi, while curious about how Elizabeth is able to stay on the lamppost, decides to work on a more practical problem and compute how fast Elizabeth’s shadow is shrinking. How fast is the shadow shrinking when Elizabeth is on her way down and reaches the ground.

5. a. After realizing they aren’t actually IN a biology class, and therefore don’t NEED to study for the test, Sid and Tom are using their windfall free time by painting the walls of the math lounge Navy and Gold. Sid is standing on top of a two-piece extension ladder leaning against the wall when Jill walks by. Jill becomes upset that Sid and Tom are covering up her favorite orange and yellow paisley wallpaper, so she kicks the base of the ladder. Suddenly, the ladder starts collapsing at the rate of 2 feet per second AND, at the same time, its base starts sliding away from the wall at the rate of 3 feet per second. How fast is Sid falling (the top of the ladder moving down the wall) when he is 8 feet from the ground and the base is 6 feet from the wall?

b. Anne and Kim live next to Bart who has a very loud stereo. The volume knob goes to 11, turning half a circle (angles \( \theta \) between \( 0^\circ \) and \( 180^\circ \) ). The volume of the music, usually Lil Nas X, is given by the function \( V(\theta) = 110 \sin\left(\frac{\theta}{2}\right) \) decibels (dB). One night at 3:30 in the morning they hear “gonna take my horse to the old town road” increasing from a volume of 88 dB at a rate of 1 decibel per second! At what rate can Anne and Kim deduce that Bart is cranking the volume knob?
6. a. Tom is tired and decides to turn in from a late-night calculus study session for the Chapter 4 exam. Carol is determined to keep everyone working and has set up a searchlight 30 yards straight out from one corner of ROSS Hall. Jon starts running from that corner along the wall and Carol keeps the spotlight trained on him. When he is 50 yards down the wall, Carol is turning the spotlight at a rate of 0.07 radians per second. How fast is Tom running at that point?

b. Art is trying to fill his Pocahontas-themed play pool with a hose at a rate of 0.06 m$^3$/min. The top of the pool is 2 m by 2 m square, the “beach entry” has zero depth along one side, and the “deep end” is 0.5 m at the opposite side with a constant slope in between. Art is unaware that Jessie has drilled a small hole in the deep end to drain water while he tries to fill it. When the water is half way to the top of the deep end, Art notices the depth there only seems to be rising at 0.02 m/min. At what rate should he infer water is draining out of the hole?