Lab 13: Definite Integrals – Part 2

Work with your group on the context assigned to you. We encourage you to collaborate both in and out of class, but you must write up your responses individually.

Context 1: Sam is tired of walking up two flights of stairs to get to calculus class every day, so he enlists Kelli to help him build a giant spring to lift him perfectly up to the second floor window. They order a two-story tall spring from Katelyn’s Giant Spring Limited Liability Co. When it arrives, it is packaged already compressed down 5 m shorter than its resting length. They figure they need to compress it another 5 m in order to climb on from ground level before launch. Tony walks by and points out this will take a lot of energy, saying:

For a constant force \( F \) to move an object a distance \( d \) requires an amount of energy** equal to \( E = Fd \). Hooke’s Law says that the force exerted by a spring displaced by a distance \( x \) from its resting length (compressed or stretched) is equal to \( F = kx \), where \( k \) is a constant that depends on the particular spring.

*The standard unit of force is Newtons (N), where 1 N = 1 kg \( \cdot \) m/s\(^2\) or the force required to accelerate a 1 kg mass at 1 m/s\(^2\). Increasing either the mass or the acceleration rate therefore requires a proportional increase in force.

**The standard unit of energy is Joules (J), where 1 J = 1 N \( \cdot \) m or the energy required to move an object with a constant force of 1 N a distance of 1 m. Increasing either the force or the distance requires a proportional increase in energy.

Sam and Kelli’s spring has a spring constant of \( k = 155 \text{ N/m} \).

Lab Preparation:

a. Use the Geogebra graph for your context to determine how many subdivisions are required to find an approximation accurate to within 30 Joules.

b. Using the Geogebra graph, how many times greater is the energy required to compress the spring from 7.5 m to 10 m shorter than its natural length than it takes to stretch the spring from 5 m to 7.5 m?

Lab:

1. Find an approximation accurate to within 0.5 Joules.
2. Write a formula indicating how to find an approximation accurate to within any pre-determined error bound, \( \varepsilon \).
3. Write a definite integral expressing the exact amount of energy required to stretch the spring.
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Context 2: Oh no! Chris accidentally broke Horsetooth Dam! Specifically, they cracked it loose from the canyon walls and floor, leaving nothing to hold back the tremendous force of water in the reservoir. Luckily, Erin is thinking quickly and braces herself against the dam to hold back the impending flood. Josh decides he should figure out exactly how much force Erin needs to exert to hold up the dam. Luckily Jarrod is able to provide the key information to figure this out:

A uniform pressure $P^{**}$ applied across a surface area $A$ creates a total force* of $F = PA$. The density of water is 1000 kg per cubic meter, so that under water the pressure varies according to depth, $d$, as $P = 9800d$. In this activity you will approximate the total force of the water exerted on a dam 63.26 meters wide and extending 25 meters under water.

*The standard unit of force is Newtons (N), where 1 N = 1 kg·m/s² or the force required to accelerate a 1 kg mass at 1 m/s². Increasing either the mass or the acceleration rate therefore requires a proportional increase in force.

**Pressure is the force per unit area, $P = F/A$, so for example a force of 6 N applied over a 2 m² area would generate a pressure of 3 N/m². Increasing the force would increase the pressure proportionally. Increasing the area would decrease the pressure proportionally (an inverse proportion).

Lab Preparation:

a. Use the Geogebra graph for your context to determine how many subdivisions are required to find an approximation accurate to within 2,000,000 N.

b. Using the Geogebra graph, how many times greater is the force of the water on the top half of the dam than on the bottom half?

Lab:

1. Find an approximation accurate to within 50,000 N.

2. Write a formula indicating how to find an approximation with any pre-determined error bound, $\varepsilon$.

3. Write a definite integral expressing the exact force of the water on the dam.
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Context 3: Aliens have captured Shawn! He will be set free only if he can beat their “hero” at a game of Gorgo. The game is simple. A large metal bar 10 meters long (Metric aliens. Who knew?) is brought before the contestants. Whoever can guess the mass of the bar most precisely, wins.

When the day of the contest arrives, Shawn is shoved into a huge arena before millions of screaming aliens. While staring at the long metallic Gorgo bar, he hears a low rumble begin, and a fearful hush falls over the crowd. As a door at the opposite end of the arena lifts, Shawn gasps in horror: it’s Elvis.

Elvis looks at the bar for only a second, mutters something unintelligible, and the crowd breaks out into a delirium, throwing Twinkies into the arena, which Elvis collects before disappearing back through the doorway. Now it is Shawn’s turn. Inspecting the Gorgo bar, he notices it is stamped,

Tom and Ann’s Fine Gorgo Bars: Our precision crafting process guarantees a perfectly uniform diameter of 10 cm. We carefully blend only the purest metals to ensure its density increases at a constant rate from 4.2 grams per cubic centimeter at one end to 33.8 grams per cubic centimeter at the other. We hope this hand-crafted Gorgo bar will provide centuries of entertainment for you and your family.

Oh… Eric just sent Shawn a text reading “mass of obj w const dnsty d & vol v is $M = dv$”. Thanks, Eric!

Lab Preparation:

a. Use the Geogebra graph for your context to determine how many subdivisions are required to find an approximation accurate to within 17,000 grams.

b. Using the Geogebra graph, how many times more massive is the denser half of the pole than other half?

Lab:

1. Find an approximation accurate to within 300 grams.

2. Write a formula indicating how to find an approximation with any pre-determined error bound, $\varepsilon$.

3. Write a definite integral expressing the exact mass of the pole.
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**Context 4:** Tiffany was on a walk Friday evening, basking in the satisfying afterglow of another calculus test. Suddenly an unknown glowing substance fell on the ground in front of her, presumably from outer space. She took the substance to Mark and Bill who decided it would make the perfect liquid core for their new invention, the Happy Fun Ball. They want to put all of the substance in, but Regis reminded them what happened the last time they filled the 1-foot radius Happy Fun Ball past 21.7 inches, and nobody wanted that to happen again! Needing to know exactly how much substance to pour in, James reminded them,

\[
V = Ah.
\]

Since they can easily compute the volume of the bottom half of the sphere, they all decide to focus on approximating the volume contained in the remaining 9.7 inches.

**Lab Preparation:**

a. Use the Geogebra graph for your context to determine how many subdivisions are required to find an approximation accurate to within 15 in\(^3\).

b. Using the Geogebra graph, how many times greater is the volume in the first 4.85 inches (above the middle of the sphere) than the volume in the next 4.85 inches?

**Lab:**

1. Find an approximation accurate to within 0.37 in\(^3\).

2. Write a formula indicating how to find an approximation with any pre-determined error bound, \(\epsilon\).

3. Write a definite integral expressing the exact volume of water in the bottle.
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Context 5: Adriana has invented a way of measuring the difficulty of calculus problems, $a$, in units of “adrii” on a scale $0 \leq a < \infty$. (The existence of a problem with difficulty $a = \infty$ adrii is a matter of contentious debate.) Joe managed to trick Dr. Oehrtman into revealing that the Chapter 5 exam will only have problems with difficulty greater than 1 adrii (harder than precalculus) and easier than 5 adrii (easier than Dr. Oehrtman can do in 50 minutes). Dana has a uniform distribution for the proportion of her brain that can think about problems of difficulty up to 4 adrii, specifically the probability density is

$$D(a) = \begin{cases} 0.25, & 0 \leq a \leq 4 \\ 0, & a > 4. \end{cases}$$

This means we can just multiply to determine the proportion of her brain that can think about problems in any difficulty range. For example, for difficulty $1 \leq a \leq 5$, the proportion is

$$P = 0.25(4-1) + 0(5-4) = 0.75.$$ 

The probability density, $K(a)$, for the proportion of Kate’s brain that can think of problems of difficulty $a$ adrii is

$$K(a) = ae^{-a}.$$ 

Joe and Adriana make a bet whether Dana or Kate (respectively) will need to use more of their brain during the Chapter 5 exam. What proportion does Kate need? Who wins the bet?

Lab Preparation:

a. Use the Geogebra graph for your context to determine how many subdivisions are required to find an approximation accurate to within 1% (that is the proportion should be within 0.01).

b. Using the Geogebra graph, how many times more of Katie’s brain can think of problems between 1 and 3 adrii than is able to think of problems between 3 and 5 adrii?

Lab:

1. Find an approximation accurate to within 0.01725% (that is the proportion should be within 0.0001725).

2. Write a formula indicating how to find an approximation with any pre-determined error bound, $\varepsilon$.

3. Write a definite integral expressing the exact proportion of Katie’s brain that can think of problems of difficulty $1 \leq a \leq 5$. 

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Challenge Context: Fluid traveling at a velocity $v$ across a surface area $A$ produces a flow rate of $F = vA$. Poiseuille’s law says that in a pipe of radius $R$, the viscosity of a fluid causes the velocity to decrease from a maximum at the center ($r = 0$) to zero at the sides ($r = R$) according to the function $v = v_{\text{max}} \left(1 - \frac{r^2}{R^2}\right)$. In this activity you will approximate the rate that water flows in a 4-inch diameter pipe if $v_{\text{max}} = 4.44$ ft/s.

Lab Preparation:

a. Use the Geogebra graph for your context to determine how many subdivisions are required to find an approximation accurate to within 0.05 cfs.

b. Using the Geogebra graph, how many times greater is the flow rate in the center half area of the pipe ($r = 0$ inches to $r = 1.4$ inches) than in the outer half area of the pipe ($r = 1.4$ inches to $r = 2$ inches)?

Lab:

1. Find an approximation accurate to within 0.0001 cfs.

2. Write a formula indicating how to find an approximation with any pre-determined error bound, $\varepsilon$.

3. Write a definite integral expressing the exact flow rate in the pipe.