Lab 1: Calculus I Review
Rate of Change and Accumulation

**Lab Preparation:** Answer the following questions *individually* and bring your write-up to class.

Graph $P(t) = \frac{1}{1 + e^{-t}}$ on your calculator.

1. Derivatives:
   a. Explain in words the graphical meaning of $P'(2)$.
   b. Recall the definition of the derivative, $P'(t) = \lim_{\Delta t \to 0} \frac{P(t + \Delta t) - P(t)}{\Delta t}$.
      Find both an underestimate and an overestimate for $P'(2)$.
   c. Explain in words the graphical meaning of your answers to Part b.
   d. How do you know your answers to Part b are under or over estimates?

2. Definite Integrals:
   a. Explain in words the graphical meaning of $\int_{0}^{4} P(t) \, dt$.
   b. Recall the definition of the definite integral, $\int_{a}^{b} P(t) \, dt = \lim_{n \to \infty} \sum_{i=1}^{n} P(t_i) \Delta t$.
      Find both an underestimate and an overestimate for $\int_{0}^{4} P(t) \, dt$.
   c. Explain in words the graphical meaning of your answers to Part b.
   d. How do you know your answers to Part b are under or over estimates?

3. What questions do you have about derivatives and definite integrals from your Calculus I course?
**Lab Questions**

**Instructions:** Work with your group on the problem assigned to you. We encourage you to collaborate both in and out of class, but you must write up your responses *individually*. Your work must be neat and include sufficient exposition to make the solution clear to another student who has not seen the assignment (for example, a sequence of equations without explanation will most likely receive zero credit). Pay particular attention to places where explanations using multiple representations are requested, and *explicitly* discuss the connections between your explanations using different representations. Type or write all of your work *legibly* on 8½"×11" paper with no spiral fringe, at least one-inch margins on all sides free of writing except your name, date, and assignment number, and staple all pages together.

For the following questions, let \( P(t) = \frac{1}{1 + e^{-t}} \).

1. **Derivatives:**
   a. Using \( \Delta t = \pm 0.5 \) in the definition of the derivative, find both an underestimate and an overestimate for \( P'(2) \). What is a bound on the error for these approximations?
   b. On a full-page graph, clearly label \( P'(2) \), your underestimate, your overestimate, the error for each approximation, and the error bound.
      *(Use an appropriate scale to best represent your answers!)*
   c. Find an underestimate and overestimate to approximate \( P'(2) \) accurate to within 0.001.
   d. Use differentiation rules to compute the exact value of \( P'(2) \).
   e. What are the exact errors for your approximations in Parts a and c? Are the errors smaller than the corresponding error bounds?

2. **Definite Integrals:**
   a. Using \( \Delta t = 0.5 \) in the definition of the definite integral, find both an underestimate and an overestimate for \( \int_0^4 P(t) \, dt \). What is a bound on the error for these approximations?
   b. On a *new* full-page graph, clearly label \( \int_0^4 P(t) \, dt \), your underestimate, your overestimate, the error for each approximation, and the error bound.
      *(Use an appropriate scale to best represent your answers!)*
   c. Find an underestimate and overestimate to approximate \( \int_0^4 P(t) \, dt \) accurate to within 0.05.
   d. Show that \( \frac{d}{dt} \ln(e^t + 1) = P(t) \) and use this to find the exact value of \( \int_0^4 P(t) \, dt \).
   e. What is the exact error for your approximations in Parts a and c? Are they smaller than the corresponding error bounds?