Lab 4: Arclength – Part 1

Instructions: Work with your group on all questions in this lab. We encourage you to collaborate both in and out of class, but you must write up your responses individually. Your work must be neat and include sufficient exposition to make the solution clear to another student who has not seen the assignment (for example, a sequence of equations without explanation will most likely receive zero credit). Pay particular attention to places where explanations using multiple representations are requested, and explicitly discuss the connections between your explanations using different representations. Draw a picture of a torus in the lower right corner of your first page for five extra credit points. Type or write all of your work legibly on 8½"×11" paper with no spiral fringe, at least one-inch margins on all sides free of writing except your name, date, and assignment number, and staple all pages together.

1. Draw a full page graph showing:
   a. The graph of \( f(x) = \sin x \) between \( x = 0 \) and \( x = \frac{\pi}{2} \).
   b. Points on the graph for \( x = 0, x = \frac{\pi}{6}, x = \frac{\pi}{3}, \) and \( x = \frac{\pi}{2} \) labeled with the exact values of their \((x, y)\)-coordinates. Note that this partitions the domain into 3 equal subintervals.
   c. Three secant lines connecting the points in Part b labeled with their lengths.

2. The sum of the lengths of the three secant lines from Question 1 can be used to approximate the arclength of the graph of \( f(x) = \sin x \) between \( x = 0 \) and \( x = \frac{\pi}{2} \).
   a. What is this approximation?
   b. Is it an underestimate or overestimate? Explain how you know.

3. Find a way to compute the approximation in Question 2 using a \texttt{sum(seq(...}) command. You will then generalize this command to other partitions of the interval \([0, \frac{\pi}{2}]\).
   a. Write down your \texttt{sum(seq(...} command for \( n = 3, \Delta x = \frac{\pi}{6} \) and verify that it works by making sure it gives the same answer you got in Question 2.
   b. Modify and evaluate your \texttt{sum(seq(...} command for \( n = 4, \Delta x = \frac{\pi}{5} \).
   c. Modify and evaluate your \texttt{sum(seq(...} command for \( n = 50, \Delta x = \frac{\pi}{100} \).
   d. Modify and evaluate your \texttt{sum(seq(...} command for \( n = 500, \Delta x = \frac{\pi}{1000} \).